COMMENTS ON SUTHERLAND'S *KANT'S MATHEMATICAL WORLD*

Tyke Nunez, Pacific APA, April 6th, 2023.

In his book, Daniel Sutherland encourages his reader to rethink Kant's philosophy of mathematics, and in light of this, to also rethink Kant's account of empirical cognition. At the heart of Sutherland's project is a reassessment of the role of magnitudes in Kant's philosophy. Kant explains the concept of magnitude (quantum) as "the consciousness of the homogeneous manifold in intuition in general, insofar as through it the representation of an object first becomes possible" (B203). My comments will focus on the role of this concept of magnitude in Sutherland's account. I'm a big fan of, especially, the second half of this book and I'll begin by pointing out why I love chapter seven, before posing a question that I was left with after reading the later chapters. Then I will turn to the earlier sections of the book and propose a friendly amendment to Sutherland's account concerning the role of construction and the relation of the categories of quantity to intuition.

§1 – Homogeneity and why intuition is fundamental to mathematics

Chapters 7 & 8 of the book offer the deepest reflection I know of on the distinctive structure of intuition. I take the alpha of Kant's theoretical philosophy to be his division between sensibility and understanding. Because these chapters make significant progress on exactly why intuition makes an irreducibly non-conceptual contribution to cognition, I take them to bring our understanding of the origins of Kant's theoretical philosophy forward in a significant and fundamental way. Thus, if you take anything away from these comments, I hope that it is that chapters seven and eight—and really chapters four through nine—of Sutherland's book are well worth reading and rereading.

To convince you, let me say a little bit about what Sutherland does in chapter seven. Here Sutherland's focus is on the notion of homogeneity. He distinguishes mere logical homogeneity from what he calls 'strict homogeneity.' Two things are logically homogeneous in some respect when they fall under the same concept. For example, a dog and a horse are logically homogeneous with respect to the concept <mammal>. Two things are strictly homogeneous, however, when they are completely logically homogenous and there is no qualitative difference between them. This is the kind of homogeneity Kant has in mind when he explains the concept of magnitude as "the consciousness of the homogeneous manifold in intuition in general, insofar as through it the representation of an object first becomes possible" (B203). With strict homogeneity, we have two things that will fall under all of the same concepts and are qualitatively identical yet are numerically distinct. Examples of this kind of

bare numerical difference include two identical one-inch cubes, two identical raindrops, and two drops of water on two needle points. As Sutherland points out, "the representation of strict homogeneity [or bare numerical difference] requires something extra-logical in the sense that it requires something beyond the reach of conceptual representation on its own" (p. 201). And a basic task of intuition is to provide this kind of extra-logical knowledge of numerical difference.

While laying out how intuition performs this task, Sutherland also argues that Kant takes the representation of such bare numerical difference to be foundational to mathematics. To get a sense of why, note first that, according to Kant, "a part of space, even though it might be completely similar and equal to another, is nevertheless outside of it, and is on that account a different part from that which is added to it in order to constitute a larger space" (A264/B320). Furthermore, Kant claims that "according to mere concepts of the understanding, it is a contradiction to think of two things outside of each other that are nevertheless fully identical in respect of all their inner determinations (of quantity and quality)." This is because the understanding only represents a thing through its inner determinations, and so if these determinations are identical than "it is always one and the same thing thought twice (numerically one)" (20:280). Thus, because Kant holds that the composition of strictly homogeneous parts into ever larger wholes is characteristic of "everything that can be considered mathematically" (B201n), and such homogeneous parts are fully identical with respect to their inner determinations, the representation of bare numerical difference that intuition makes possible is fundamental to mathematics. This, Sutherland argues, is the basic reason why intuition is the foundation of all of mathematics on Kant's account.

§2 – Why couldn't concept's do the work of intuitions, according to Sutherland?

To put my critic's hat on, by the end of chapter seven I was left with a question. Suppose that we have a view, which is like some readings of Aristotle's or Leibniz's, on which the attributes of mathematical objects are qualities in the sense of specific differences of the objects. For example, suppose it is a specific difference of this cube that the length of its diagonal is root three times the length of one of its sides, or it is a specific difference of, say, 12 that it equals 5 +7. On this view, since the qualities of mathematical objects are specific differences, they are conceptualizable. And since these are conceptualizable features, then it looks like mathematical objects can be distinguished conceptually. Now, why exactly is Kant justified in rejecting this kind of view on Sutherland's reading? Why can't we treat 'equaling 5 +7' as a quality of 12?

Or to put this in a more Kantian parlance, Kant would agree that 12's equaling 5 +7, or a triangle's angles adding up to 180 degrees, are synthetic predicates of their objects, it's just that he would reject the claim that they are analytic predicates of them. What separates these two kinds of predicates on Sutherland's reading? And what reasons does Sutherland's Kant have for insisting against Leibniz and Aristotle that this separation is fundamental?

We can also come at this issue through intuition. On Kant's view we get *a priori* portions of space and time that are identical but distinct. But why is this? He maintains that the wholes of space and time are prior to their parts. As a result, it looks like each in-itself identical part of space and time will have properties that are specific to it. So like how the number 12 has properties that look specific to it, which would differentiate it from all of the other numbers, it looks like each part of the one whole space will have properties that are specific to it, which seem like they would be adequate to differentiate the parts of space (and time) from one another. For this reason, it looks like the specific differences of numbers, as well as the parts of space and time, should be sufficient to conceptually distinguish the numbers or the parts of space and time. But if that were the case then it seems like concepts would be able to do the work of intuition, and we could jettison intuition. So what is it, on Sutherland's reading of Kant, that blocks this kind of move?

I suspect that at least part of Sutherland's answer will be that only intuitions can represent singularly in the sense developed in his fifth chapter. That shifts the bump under the rug, but it shifts it in a fruitful way. Now we can ask, "why is representing singularly so important? And why must we distinguish it from the general representing distinctive of concepts?"

These questions cut to the heart of the motivation for the Kantian distinction between intuition and concept, and thus for the starting point of Kant's theoretical philosophy. I don't know of anyone who has given these questions a satisfactory answer, and thus I don't know of anyone who has seen to the core of this dimension of Kant's project. (Although of course many people—and especially Lanier—have made progress on them.) Still, I think coming at these questions through Daniel's book in the way that I just have lets us ask them in a newly clear-eyed way and it should reveal a fundamental and unappreciated motivation for his radically un-Leibnizian innovation of distinguishing in-kind intuitions from concepts.

§3 – Friendly amendment: two *ways* the concept of quantity determines intuition

I'd like to turn to the earlier chapters of the book, where one focus of Sutherland's discussion is on the relationship between our concepts and intuitions of magnitudes, and I'd like to offer a friendly amendment to Sutherland's view that I think can clear up some difficulties.

In the first chapter (§1.3, p. 16), Sutherland contrasts the interpretation he will develop with one that attempts to separate the conditions of the possibility of experience from the conditions of the possibility of the exact sciences. As he puts it there, I agree that these two sets of conditions are intertwined, and I think he is right to think that Kant conceives of the exact sciences as a systematic development of the same knowledge that we have first in everyday experience. Nonetheless, as I reached the end of chapter two, I came to worry that Sutherland was seeing the conditions of the possibility of experience and the conditions of the possibility of the exact sciences as more intertwined than I take them to be for Kant.

My worry stems from how Sutherland is thinking about the construction of the concept of quantity and the role that he sees the categories of quantity as playing in empirical perception. The core example of how the category of quantity is involved in this kind of perception for Kant comes from §26, when he discusses the apprehension of a house. Here there seemed to be some tension in Sutherland's discussion. On the one hand, when he first introduces the case of the apprehension of the house, he seems to acknowledge that Kant does not think we construct the shape of the house, because he points out how Kant says here that I only "*as it were* draw it's shape…" (p. 44 n30). On the other hand, as Sutherland goes on, he suggests that the empirical apprehension of the house not only involves merely '*as it were'* drawing its shape, but actually *constructing* this shape (p. 47 n36). Furthermore, he claims that "the cognition of a determinate space or time requires construction in intuition" (p. 45 n34). So, although I'm not sure that on his considered view Sutherland thinks we construct the shape of the house in apprehending it, there were a few passages that gave this impression.

I do not think Kant holds that we perform a construction in empirical apprehension. Why? The construction of concepts happens through an act of the will; the concepts of mathematics are also electively made (A729/B757). When I empirically apprehend a house, my will is not involved. It is not even involved in my 'as it were' drawing its shape. Furthermore, as Sutherland points out, "Kant delimits the field of mathematics by appeal to construction" because mathematics "consists of those cognitions established through the construction of concepts, and only those cognitions" (p. 47). But Kant nowhere suggests that empirical perception is a part of mathematics or that such perception involves construction.

Still, I have an idea about why Sutherland sometimes thinks we must be performing constructions in everyday empirical apprehension, as in the case of seeing a house. At the beginning of the Axioms of Intuition Kant claims that appearances cannot be apprehended "except through the synthesis of the manifold through which the representations of a determinate space or time are generated, i.e., through the composition of that which is homogeneous" (B202-203). Of course, we've seen that in mathematics we compose that which is homogeneous, so it looks like the same kind of act of synthesis is involved in empirical apprehension. Indeed, if we consider, with Kant, the cases of constructing a circle in *a priori* intuition and the empirical apprehension of a circular plate (A137/B176), we might say that the very same act of synthesis is involved in the two cases, since in both the homogeneous manifold is composed into a circle. That is, in both cases the composed homogeneous manifold of intuition is equidistant from a center point.

Nonetheless, while it is true in this sense that the same synthesis is involved in both the determinate construction of a circle and the determinate empirical apprehension of a circular plate, in the former case we make the *a priori* intuition through an elective (*willkürlich*) act of construction, while in the latter case we are given the empirical intuition and we determine it through, *as it were*, drawing the circle in apprehension, which is not something that we choose electively to do. Thus, we must be careful about the sense in which it is 'the same synthesis' that is involved in the made mathematical or the given empirical cases. In both cases there is a determination through the category of quantity, and both are species of composition, but the cases are separate because of the quite different role of the will in each. My friendly amendment to Sutherland's reading is that he should understand the sense in which the same synthesis is involved in both cases in this way, and thus should not take construction to be involved in the case of empirical apprehension.

§4 – 'Indeterminate intuition of a whole' and a footnote from the First Antinomy

In these same passages, Sutherland also makes clear that he thinks we can have an intuition of a whole within boundaries, like of a spatial figure, although that intuition is indeterminate (p. 45 n34, p. 85 n41). I'm not clear on what it would mean to have such an intuition on Sutherland's account. But before I get into why, let me first say that it is in his discussions of the possibility of indeterminate intuitions within boundaries that Sutherland comes closest to taking a stand on issues of direct relevance to the conceptualism/non-conceptualism debate. In this connection, Sutherland is at pains to stay neutral, and consistently draws his reader's attention away from the debate and back to issues that Kant is more clearly concerned with. I hope I don't get into to much trouble for saying so, but I

like this attitude towards the dispute. I tend to find both conceptualism and non-conceptualism to be infused with contemporary ways of thinking about perception. I suspect that it is in part because the concerns of the debate really stem from this contemporary discussion, not from Kant himself, that although this debate has attracted a lot of attention from many of the most brilliant minds working on Kant today, the back and forth has generated a whole lot more heat than it has light. As a result, I find Sutherland's attempts at neutrality and redirection laudable.

So why can't I get clear on how an indeterminate intuition of a whole within boundaries is possible on Sutherland's view? He argues that there are indeterminate and determinate magnitudes (p. 84-85), and that determinate magnitudes divide into extensive and intensive magnitudes (p. 55, 96). In chapter four, he solves what he calls the extensive magnitude regress problem by pointing out that although intuitions of extensive magnitudes—like line segments—start and end at determinate boundaries—like points—in between they are continuous and have no smallest parts, and so their parts are indeterminate (p. 101, p. 119). In this sense, the line segment is determinate because it has two end points, but the continuum connecting them is an indeterminate magnitude because it could be indefinitely divided. Here it seems that to be determinate means to be a whole and have boundaries, and to be indeterminate is not to have boundaries. Insofar as we think only of the indeterminate portion of our line segment, on Sutherland's reading, we leave out its boundaries and it is not a whole. Thus, on Sutherland's view, it seems to me that there cannot be an *indeterminate whole* intuition within boundaries. This looks like a *contradicto in adjecto*, as Kant would say.

Sutherland is pushed to accept the possibility of indeterminate wholes enclosed within boundaries by a footnote in the First Antinomy (A462n/B454n). I think if he takes on my friendly amendment and allows that the categories of quantity are deployed in two ways, one in *a priori* construction, the other in empirical apprehension, then he can read this important footnote in a different way. In the text, Kant says:

> We can think of the magnitude of a *quantum* that is not given as within certain boundaries of every intuition* in no other way than by the synthesis of its parts, and we can think of the totality of such a *quantum* only through the completed synthesis, or through the repeated addition of units to each other.† (A427/B455-A428/B456)
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> Nun können wir die Größe eines Quanti, welches nicht innerhalb gewisser Grenzen jeder Anschauung gegeben wird,* auf keine andere Art, als nur durch die Synthesis der Theile und die Totalität eines solchen Quanti nur durch die vollendete Synthesis, oder durch wiederholte Hinzusetzung der Einheit zu sich selbst gedenken.*

The two attached footnotes are:

> * We can intuit an indeterminate *quantum* as a whole, if it is enclosed within boundaries, without needing to construct its totality through measurement, i.e., through the successive synthesis of its parts. For the boundaries already determine its completeness by cutting off anything further.

> * Wir können ein unbestimmtes Quantum als ein Ganzes anschauen, wenn es in Grenzen eingeschlossen ist, ohne die Totalität desselben durch Messung, d.i. die successive Synthesis seiner Theile, construiren zu dürfen. Denn die Grenzen bestimmen schon die Vollständigkeit, indem sie alles Mehrere abschneiden.
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> † The concept of a totality is in this case nothing other than the representation of the completed synthesis of its parts, because, since we cannot draw the concept from an intuition of the whole (which is impossible in this case), we can grasp it, at least in the idea, only through the synthesis of the parts up to their completion in the infinite.
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> † Der Begriff der Totalität ist in diesem Falle nichts anderes, als die Vorstellung der vollendeten Synthesis seiner Theile, weil, da wir nicht von der Anschauung des Ganzen (als welche in diesem Falle unmöglich ist) den Begriff abziehen können, wir diesen nur durch die Synthesis der Theile bis zur Vollendung des Unendlichen wenigstens in der Idee fassen können

These are complex passages where Kant is differentiating how we think of an infinite magnitude like space and time from how we think about their parts. Sutherland's focus is on the first star footnote (p. 45 n34; p. 102ff). He rightly points out that the focus in that note is on the construction of the *quantitas* of a *quantum* through measurement. This involves taking a plurality of units together as a totality, and thus an *a priori* mathematical construction that deploys all three categories of quantity. Sutherland rightly reads the first footnote as pointing out that in order to intuit a quantum as a whole within boundaries we do not need to perform such an *a priori* construction.

Now, in line with my friendly amendment, once we recognize that the categories of quantity have two kinds of deployment—one in made *a priori* construction, the other in given empirical apprehension—a new contrast is opened. Rather than reading Kant as affirming the possibility of a completely indeterminate intuiting of a whole within boundaries, it looks like the sense in which the intuition is indeterminate is that it has not been measured through an *a priori* construction. That does not entail that it hasn't been empirically apprehended. After all, it is a whole enclosed within boundaries. This gives us a way of making sense of how Kant could think that we can cognize an extensive *quantum* within boundaries, apart from determining its measure (*quantitas)* (compare Sutherland's discussion, p. 120), even though enclosure within boundaries, on Sutherland's reading, seems to require the involvement of the categories of quantity, because to have such enclosure is to have a determinate intuition (p. 43-45). On this reading of the footnote, then, the apprehension of the intuition within boundaries involves the deployment of the categories of quantity in empirical apprehension, while this is nonetheless indeterminate in so far as we have not constructed its measure *a priori* through those same categories. Thus, keeping the amendment in mind, Sutherland is not saddled with the supposition of a completely *indeterminate whole* intuition.

This point may also ameliorate some lingering large-scale discomfort that a reasonable reader might be left with by Daniel's interpretation. The Axioms of Intuition, according to Kant, have the following principle in the 1st and 2nd editions of the first Critique, respectively:

> All appearances are, as regards their intuition, extensive magnitudes. (A162)
>
> All intuitions are extensive magnitudes. (B202)

According to Daniel, however,

> A more precise statement of the A-edition principle would be: All appearances are, as regards their **determinate** intuition, extensive ***quanta***. A corresponding more precise statement of the B-edition formulation would be: All **determinate** intuitions are extensive ***quanta***. (p. 93, my emphasis)

If Daniel were to adopt my friendly amendment, however, then there would be a clear reason for Kant not to have said "determinate intuition" in either principle. After all, "determinate" intuition might be read as implying that the quanta had been *constructed* mathematically. On my amendment this is what Kant means in the First Antinomy by a determinate quantum. Such construction, however, is absent with mere perception as of the house because there we are not actually constructing its measurement. So, to avoid the suggestion that determinate mathematical construction of the intuition is required, and because he could rely on the place of the Axioms of Intuition in the Transcendental Analytic to convey that the intuitions in question are nonetheless determinately apprehended, Kant had good reason not to include "determinate" in his formulation of the Axioms of Intuition principle. In this way, Daniel can avoid re-writing the principle as he does, while still holding on to the substance of the point that leads him to want to rewrite them, which is that the intuition in question cannot include the un-apprehended and indeterminate formal intuitions of space and time.