This handout can be found at: https://tykenunez.nfshost.com/HOPOS_Handout.pdf

THE TWILIGHT OF INTUITION AND RUSSELL'S EARLY HYLOMORPHISM: SPACE IN RUSSELL'S *FOUNDATIONS OF GEOMETRY*

Tyke Nunez, HOPOS June 2022.

(A forthcoming essay draft on the doctrine of internal relations can be found on my website: tykenunez.com)
(If you would like to look at a draft short book length manuscript, please email me: an16@mailbox.sc.edu)

§1 - Introduction

Russell's *An Essay on the Foundations of Geometry* (*EFG*) stands at the crossroads between a tradition tracing back to Euclid, Eudoxus, and Aristotle, through Kant, that took space to be the foundation of mathematics and the new tradition of the late 19th and 20th century that places logic at its foundation. For this reason (among others), the conception of space of *Foundations* deserves more attention.

Russell develops this conception by criticizing the views of **Kant** and Carl **Stumpf** (*EFG*, ch. 4).
Both, he claims, illicitly hypostatize empty space, and land in contradictions (§203, §207, §196-7, §131).
Russell's account in *Foundations* is a hylomorphic one. (Like Aristotle's or Kant's. *Pace* later Russell.)
Spatial figures are the "*Thises"* of geometry (§189, §199), which are a union of form and matter.
Today I would like to suggest that Russell's hylomorphism is at the core of how he resolves the antinomies that arise with the hypostatization of empty space.
I will focus on the two main antinomies he discusses: The antinomies of the point and of relations.

Empty space: *possible* spatial relations. An empty space = a "**form** of externality" (§202).
Spatial order or figures: *actual* spatial relations. Ultimately, relations between unextended atoms.

§2 – The antinomy of the point (*EFG*, §196 to §199)

Russell: Points are contradictory. Why?

> A point must be spatial, otherwise it would not fulfil the function of a spatial element; but again it must contain no space, for any finite extension is capable of further analysis. Points can never be given in intuition, which has no concern with the infinitesimal: they are a purely conceptual construction, arising out of the need of terms between which spatial relations can hold. If space be more than relativity, spatial relations must involve spatial relata; but no relata appear, until we have analyzed our spatial data down to nothing. The contradictory notion of the point, as a thing in space without spatial magnitude, is the only outcome of our search for spatial relata. This *reductio ad absurdum* surely suffices, by itself, to prove the essential relativity of space. (§196)

Points are the basic elements that compose space.
Space is extended, so its parts will be extents or figures.
Geometrical points, however, are not extended and do not contain space.
But if each one does not contain space, their collection will also not contain space.
So although points are supposed to be the basic spatial parts, they cannot be such parts.

Russell's solution: his **atoms**. On his view, spatial figures or extents consist in spatially related atoms.

> Empty space, we have said, is a possibility of diversity in relation, but spatial figures, with which Geometry necessarily deals, are the actual relations rendered possible by empty space. Our matter, therefore, must supply the terms for these relations. It must be differentiated, since such differentiation, as we have seen, is the special work of space. We must find, therefore, in our matter, that unit of differentiation, or atom, which in space we could not find. This atom must be simple, *i.e.* it must contain no real diversity; it must be a *This* not resolvable into *Thises*. Being simple, it can contain no relations within itself, and consequently, since spatial figures are mere relations, it cannot appear as a spatial figure; for every spatial figure involves some diversity of matter. But our atom must have spatial relations with other atoms, since to supply terms for these relations is its only function. It is also capable of

> having these relations, since it is differentiated from other atoms. Hence we obtain an unextended term for spatial relations, precisely of the kind we require. (§199)

These contain no "intrinsic spatial" properties (§200) and are "non-spatial simple" elements (§199).

Because they are non-spatial simples, a heap or composition of atoms does not constitute a space.

Nonetheless, spatially related, these atoms constitute spatial figures and acquire spatial properties.

That which spatially relates these atoms, is a "form of externality." It is a system of possible spatial relations. Spatial atoms are what make these relations actual.

Russell maintains that this solves the antinomy of the point. But why are atoms an improvement?

This solution is hylomorphic: geometrical figures are constituted through a form and a matter.

Non-spatially related atoms are non-related relata. They are unenformed **matter**.

Through an empty space, or a "form of externality," they become related relata — enformed matter.

Such a "form of externality" is an unenmattered geometrical form.

Spatial figures or extents — enformed matter — can always be divided into further extents — further enformed matter. One will never hit simple atoms alone, unenformed by their spatial relations.

The antinomy arose because extensionless points were supposed to compose an extension, a space.

Unlike points, however, atoms are non-spatial, so are not supposed to compose space.

And because atoms are not spatial elements, in repeatedly dividing a space, there is no expectation that one will reach atoms, as the basic elements of space.

Thus, so long as one does not take atoms to compose space, but only takes them to constitute space when they are spatially related, and one does not confuse non-spatial atoms with spatially related atoms—unenformed and enformed matter—the antinomy of the point disappears.

§3 – The antinomy of spatial relations (*EFG*, §201 to §207)

Russell puts the antinomy of spatial relations as follows:

> Spatial figures must be regarded as relations. But a relation is necessarily indivisible, while spatial figures are necessarily divisible *ad infinitum*." (*EFG*, §195, p. 189; §202, p. 194).

Here he seems to be thinking that a spatial figure, e.g., a line segment "—" is divisible into parts: "-" and "-" and that the whole line segment can be thought of as a relation between its two parts.

For this reason, a spatial figure is regarded as a relation.

Nonetheless, a relation is indivisible. Why?

Well, a relation relates relata. For example, "loves" relates in "Othello loves Desdemona." The relation "love," however, does not divide into parts. As a result, it seems spatial figures cannot be relations.

Russell's solution:

> **§207.** The apparent divisibility of the relations which constitute spatial order, then, may be explained in two ways, though these are at bottom equivalent. We may take the relation as considered in connection with empty space, in which case it becomes more than a relation; but being falsely hypostatized, it appears as a complex thing, necessarily composed of elements, which elements, however, nowhere emerge until we analyze the pseudo-thing down to nothing, and arrive at the point. In this sense, the divisibility of spatial relations is an unavoidable illusion. Or again, we may take the relation in connection with the material atoms it relates. In this case, other atoms may be imagined, differently localized by different spatial relations. If they are localized on the straight line joining two of the original atoms, this straight line appears as divided by them. But the original relation is not really divided: all that has happened is, that two or more equivalent relations have replaced it, as two compounded relations of father and son may replace the equivalent relation of grandfather and grandson. These two ways of viewing the apparent divisibility are equivalent: for empty space, in so far as it is not illusion, is a name for the aggregate of possible space-relations. To regard a figure in empty space as divided, therefore,

> means, if it means anything, to regard two or more other possible relations as substituted for it, which gives the second way of viewing the question.

Russell's solution is to argue that the "divisibility of the relations which constitute spatial order" or spatial figures is merely apparent (§207). He does this by distinguishing two senses of "relation."

With a forgivable anachronistic use of notation, take a line, *L*(__, - -), which is a relation with two argument places, indicated by '__' and '- -', that relates two atoms, *A* and *B*: *L*(*A, B*).

While *L*(*A, B*) is divisible, *L*(__, - -) is not. We often confuse *L*(*A, B*) with *L*(__, - -).

This is the source of the antinomy.

Put **hylomorphically**: in *L*(*A, B*) the atoms *A* and *B* are the matter and *L*(__, - -) is the form.

The antinomy arises from confusing enmattered form, *L*(*A, B*), with unenamttered form, *L*(__, - -). Enmattered form is divisible. But unenamttered form is not.

For example, we might imagine that the line *L*(*A, B*) was divided by another atom, *C*.

Now instead of the original line *L*(*A, B*), we have *L'*(*A, C*) and *L"*(*C, B*), which consist in the relations *L'* (__ , - -) and *L"* (__ , - -) relating the atoms, *A, C,* and *B*.

Thus, although the original relation *L* (__ , - -) may seem to have been divided into *L'* (__ , - -) and *L"*(__ , - -), because *L*(*A, B*), has been divided into *L'*(*A, C*) and *L"*(*C, B*), *L* (__ , - -) has not actually been divided.

Rather, it has been replaced by two new relations: *L'* (__ , - -) and *L"*(__ , - -), just as two instances of "__ is the father of - -" might replace "__ is the grandfather of - -" without the second relation being divided (*EFG*, §207).

So by distinguishing possible spaces from actual figures, Russell dissolves the antinomy.

§4 – Conclusion

Hypostatizing empty space, then, seems to generate both antinomies because with it one treats empty space as though it were spatial order.

This conflates enformed with unenformed matter, and enmattered with unenmattered form.

Why does Russell charge both Kant and Stumpf with hypostatization? For both this is a bit mysterious.

Kant explicitly rejects views of space that turn space into something real. (The problem with Newton)

Stumpf's basic observation is that we can never imagine a space or spatial figure without imagining it as having some color or quality, and so we can never represent space as genuinely empty. (Berkeley)