THE TWILIGHT OF INTUITION AND RUSSELL'S EARLY HYLOMORPHISM: SPACE IN RUSSELL'S *FOUNDATIONS OF GEOMETRY*

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§1 - Introduction

Russell's *An Essay on the Foundations of Geometry* (*EFG*) stands at the crossroads between a tradition tracing back to Euclid, Eudoxus, and Aristotle, through Kant, that took space to be the foundation of mathematics and the new tradition of the late 19th and 20th century that places logic at its foundation. For this reason (among others), the conception of space of *Foundations* deserves more attention.

Russell's account in *Foundations* is a hylomorphic one. (Like Aristotle's or Kant's. *Pace* later Russell.)
  Spatial figures are the *"Thises"* of geometry (§189, §199), which are a union of form and matter.
Roughly, on hylomorphic accounts there is some matter or stuff and then there is some form or order of that stuff. And the order or form cannot be just one more element or part of the matter.

Today I would like to suggest that Russell's hylomorphism is at the core of how he resolves the antinomies that arise with the hypostatization of empty space.
I will focus on the two main antinomies he discusses: The antinomies of the point and of relations.

§2 – The hypostatization of space

First, let me say a little bit about how Russell's argument in ch. 4 of *Foundations* is working.
Russell develops this conception of space by criticizing the views of **Kant** and Carl **Stumpf**.
Both, he claims, illicitly hypostatize empty space, and land in contradictions (§203, §207, §196-7, §131).
A secondary goal of the talk will be to understand why Russell charges both Kant and Stumpf with having a hypostatized, contradictory conception of space.

In many respects Russell takes his own view to be Leibnizian, and he clearly thinks that Kant's and Stumpf's views are too Newtonian.
It would make sense if he thinks this is the problem with both. But why would that be? It's mysterious.

Kant's view is more Newtonian than Russell's, but he aims to avoid Newtonian hypostatization.
Russell will define a form of externality through a founding figure. All spaces are of constant curvature.
  The founding figure fixes the curvature of the space along with its dimensionality.
But the founding figure aside, the whole infinite space is prior to the figures in space, just as on the Kantian and Newtonian accounts.
What then is the relevant difference? Why does Russell hold that Kant hypostatizes space?

In **Stumpf's** case, it can be a bit mysterious how his view is supposed to be Newtonian at all.
His basic observation is that we can never imagine a space or a spatial figure without imagining it as having some color or quality, and so we can never represent space as genuinely empty. (Berkeley)
Because we can never represent space as empty, it is unclear how he could be guilty of hypostatizing the representation of empty space.

We will see that the key to why Russell charges both Kant and Stumpf with hypostatizing space is that there is an important distinction between two notions of space that they cannot separate properly:

> **Empty space**: *possible* spatial relations. An empty space = a "**form** of externality" (§202).
> **Spatial order or figures**: *actual* spatial relations. Ultimately, relations between unextended atoms.

According to Russell, both Kant and Stumpf will be guilty of treating empty space as though it were spatial order, thereby hypostatizing it, and falling into Russell's antinomies.

§3 – The antinomy of the point (*EFG*, §196 to §199)

The first of Russell's antinomies: Points are contradictory. Why?

> A point must be spatial, otherwise it would not fulfil the function of a spatial element; but again it must contain no space, for any finite extension is capable of further analysis. Points can never be given in intuition, which has no concern with the infinitesimal: they are a purely conceptual construction, arising out of the need of terms between which spatial relations can hold. If space be more than relativity, spatial relations must involve spatial relata; but no relata appear, until we have analyzed our spatial data down to nothing. The contradictory notion of the point, as a thing in space without spatial magnitude, is the only outcome of our search for spatial relata. This *reductio ad absurdum* surely suffices, by itself, to prove the essential relativity of space. (§196)

Points are the basic elements that compose space.

Space is extended, so its parts will be extents or figures.

Geometrical points, however, are not extended and do not contain space.

But if each one does not contain space, their collection will also not contain space.

So although points are supposed to be the basic spatial parts, they cannot be such parts.

Russell's solution: his **atoms**. On his view, spatial figures or extents consist in spatially related atoms.

> Empty space, we have said, is a possibility of diversity in relation, but spatial figures, with which Geometry necessarily deals, are the actual relations rendered possible by empty space. Our matter, therefore, must supply the terms for these relations. It must be differentiated, since such differentiation, as we have seen, is the special work of space. We must find, therefore, in our matter, that unit of differentiation, or atom, which in space we could not find. This atom must be simple, *i.e.* it must contain no real diversity; it must be a *This* not resolvable into *Thises*. Being simple, it can contain no relations within itself, and consequently, since spatial figures are mere relations, it cannot appear as a spatial figure; for every spatial figure involves some diversity of matter. But our atom must have spatial relations with other atoms, since to supply terms for these relations is its only function. It is also capable of having these relations, since it is differentiated from other atoms. Hence we obtain an unextended term for spatial relations, precisely of the kind we require. (§199)

Atoms contain no "intrinsic spatial" properties (§200) and are "non-spatial simple" elements (§199).

Because they are non-spatial simples, a heap or composition of atoms does not constitute a space.

Nonetheless, spatially related, these atoms constitute spatial figures and acquire spatial properties.

That which spatially relates these atoms, is a "form of externality." It is a system of possible spatial relations. Spatial atoms are what make these relations actual. They are what are ordered spatially.

Russell maintains that this solves the antinomy of the point. But why are atoms an improvement?

This solution is hylomorphic: geometrical figures are constituted through a form and a matter.
Non-spatially related atoms are non-related relata. They are unenformed **matter**.

Through an empty space, or a "form of externality," they become related relata — enformed matter. Such a "form of externality" is an unenmattered geometrical form.

Spatial figures or extents — enformed matter — can always be divided into further extents — further enformed matter. One will never hit simple atoms alone, unenformed by their spatial relations.

The antinomy arose because extensionless points were supposed to compose an extension, a space.

Unlike points, however, atoms are non-spatial, so are not supposed to compose space.

And because atoms are not spatial elements, in repeatedly dividing a space, there is no expectation that one will reach atoms, as the basic elements of space.

Thus, so long as one does not take atoms to compose space, but only takes them to constitute space when they are spatially related, and one does not confuse non-spatial atoms with spatially related atoms—unenformed and enformed matter—the antinomy of the point disappears.

§4 – The antinomy of spatial relations (*EFG*, §201 to §207)

Russell puts his second antinomy, the antinomy of spatial relations, as follows:

> Spatial figures must be regarded as relations. But a relation is necessarily indivisible, while spatial figures are necessarily divisible *ad infinitum*." (*EFG*, §195, p. 189; §202, p. 194).

Here he seems to be thinking that a spatial figure, e.g., a line segment "—" is divisible into parts: "-" and "-" and that the whole line segment can be thought of as a relation between its two parts.

For this reason, a spatial figure is regarded as a relation.

Nonetheless, a relation is indivisible. Why?

Well, a relation relates relata. For example, "loves" relates in "Othello loves Desdemona." The relation "love," however, does not divide into parts. As a result, it seems spatial figures cannot be relations.

Russell's solution:

> §207. […] we may take the relation in connection with the material atoms it relates. In this case, other atoms may be imagined, differently localized by different spatial relations. If they are localized on the straight line joining two of the original atoms, this straight line appears as divided by them. But the original relation is not really divided: all that has happened is, that two or more equivalent relations have replaced it, as two compounded relations of father and son may replace the equivalent relation of grandfather and grandson. These two ways of viewing the apparent divisibility are equivalent: for empty space, in so far as it is not illusion, is a name for the aggregate of possible space-relations. To regard a figure in empty space as divided, therefore, means, if it means anything, to regard two or more other possible relations as substituted for it, which gives the second way of viewing the question.

Russell's solution is to argue that the "divisibility of the relations which constitute spatial order" or spatial figures is merely apparent. He does this by distinguishing two senses of "relation."

With a forgivable anachronistic use of notation, take a line segment, $L($__$, $- -$)$, which is a relation with two argument places, indicated by '__' and '- -', that relates two atoms, A and B: $L(A, B)$.

While $L(A, B)$ is divisible, $L($__$, $- -$)$ is not. We often confuse $L(A, B)$ with $L($__$, $- -$)$.

This is the source of the antinomy.

Put **hylomorphically**: in $L(A, B)$ the atoms A and B are the matter and $L($__$, $- -$)$ is the form.

The antinomy arises from confusing enmattered form, $L(A, B)$, with unenamttered form, $L($__$, $- -$)$. Enmattered form is divisible. But unenamttered form is not.

For example, we might imagine that the line segment $L(A, B)$ was divided by another atom, C.

Now instead of the original line segment $L(A, B)$, we have $L'(A, C)$ and $L''(C, B)$, which consist in the relations $L'($__$, $- -$)$ and $L''($__$, $- -$)$ relating the atoms, A, C, and B.

Thus, although the original relation $L($__$, $- -$)$ may seem to have been divided into $L'($__$, $- -$)$ and $L''($__$, $- -$)$, because $L(A, B)$, has been divided into $L'(A, C)$ and $L''(C, B)$, $L($__$, $- -$)$ has not actually been divided.

Rather, it has been replaced by two new relations: $L'($__$, $- -$)$ and $L''($__$, $- -$)$, just as two instances of "__ is the father of - -" might replace "__ is the grandfather of - -" without the second relation being divided (*EFG*, §207).

So by distinguishing possible spaces from actual figures, Russell dissolves the antinomy.

§5 – Kant's and Stumpf's hypostatizations of space

Why think, then, that both Kant and Stumpf hypostatize empty space? First, take **Stumpf**.

Stumpf's "Absolute contents:" Matt is 6' tall. Kathryn is 5'6" tall. Stumpf would take these to be intrinsic properties. These properties explain and ground Matt's being taller than Kathryn.

Stumpf holds every space has a center point. Every other point has coordinates in relation to it. All of the relations between points are explained and grounded in these coordinates.

Absolute contents are the collection of intrinsic properties of things that ground their relations.

He takes the collection of coordinates of points in a space to be an absolute content that grounds all spatial relations. And he (with Russell) takes spatial figures to be spatial relations.

So why does Russell think Stumpf is committed to hypostatizing empty space?

Stumpf rejects hylomorphism, so in a way it is unsurprising that he does not distinguish, say, empty space and spatial order as Russell does. But I think Russell's critique is deeper.

Russell takes Stumpf's system of spatial points or coordinates to be a conception of empty space.

This empty space completely determines the geometrical properties of the figures in that space.

As a result, Russell holds that from the point of view of geometry and its epistemology, there is no significant difference between this empty space and the order of things in space.

And because there is no difference in geometry, Stumpf hypostatizes empty space.

(Because Stumpf requires an origin point and coordinates, he also requires that every space include a conception of absolute distance, which is the substantive problem with Newton.)

Kant does not endorse Stumpf's origin points or a conception of absolute distance, and he clearly distinguishes empty space from the order of things in space. So Stumpf's problem doesn't apply.

What, then, is wrong with Kant's view according to Russell? A clue can be found in the difference between Russell and Kant I mentioned above:

Kant does not define spaces through an original figure, while Russell does.

Russell holds that without this, without "some reference to matter" (§71), geometry is impossible:

> The reference to matter is necessitated by the homogeneity of empty space. For so long as we leave matter out of account, one position is perfectly indistinguishable from another, and a science of relations is impossible. Indeed, before spatial relations can arise at all, the homogeneity of empty space must be destroyed, and this destruction must be effected by matter.

This matter that destroys the homogeneity of space are Russell's atoms. They are abstracted from actual physical atoms, although geometry is independent of physics (§69).

Because Kant does not require such matter for the differentiation of space, and is simply willing to treat empty space as given *a priori,* he would treat empty space as a given formal intuition.

Russell, however, thinks "empty space is wholly conceptual; spatial order alone is immediately experienced" (§207).

So because for Kant *a priori* empty space is prior to anything in it, material atoms do not differentiate actual space from empty space, and empty space is an intuition and not a concept, according to Russell Kant does not properly distinguish empty space from spatial order.

As a result, Russell thinks Kant treats empty space itself as too much of a thing, ala Newton.

§6 – Conclusion

Hypostatizing empty space generates both antinomies because with it one treats empty space as though it were spatial order.

This conflates enformed with unenformed matter, and enmattered with unenmattered form.

Stumpf and Kant, according to Russell, hypostatize space because they take there to be a spatial substratum that grounds spatial order.

This substratum is prior to the first things in space and forecloses properly distinguishing empty space and spatial order.